HOMEWORK 3 SOLUTIONS

6.36. Set Up: $r = 10.5 \times 10^9$ m. $T = 6.3$ days $= 5.443 \times 10^5$ s. The mass of the sun is $m_S = 1.99 \times 10^{30}$ kg.

Solve: $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{HD}}}$ so

$$m_{HD} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2(10.5 \times 10^9\ \text{m})^3}{(5.443 \times 10^5\ \text{s})^2(6.673 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 2.3 \times 10^{30}\ \text{kg};\ m_{HD} = 1.2 m_S$$

6.60. Set Up: On earth, $g_E = g = 9.80\ \text{m/s}^2$. On Mars,

$$g_M = G\frac{m_M}{R_M^2} = (6.673 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{6.42 \times 10^{23}\ \text{kg}}{(3.40 \times 10^6\ \text{m})^2} = 3.71\ \text{m/s}^2.$$

The artificial gravity acceleration is $a_{rad} = v^2/r$. $r = 400.0$ m

Solve: (a) $v = \sqrt{ra_{rad}} = \sqrt{rg} = \sqrt{(400.0\ \text{m})(9.80\ \text{m/s}^2)} = 62.6\ \text{m/s}$

(b) $v = \sqrt{rg_M} = \sqrt{(400.0\ \text{m})(3.71\ \text{m/s}^2)} = 38.5\ \text{m/s}$

(c) The free-body diagram is given in Figure 6.60. The diagram is drawn for when the rim of the station is to the right in the figure.

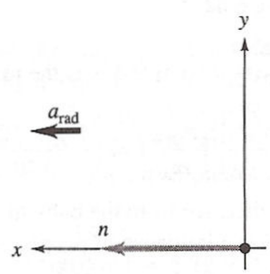

Figure 6.60

MC 2 $E_{k_{final}} - E_{k_{initial}} = -\mu_k m g s$ work energy theorem

$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 =$ $\mu_k m g = F_{friction}$

case 1 $v_f = 0$ (object comes to rest)

$$-\frac{1}{2}mv_i^2 = -\mu_k m g s$$

$$s = \frac{v_i^2}{2\mu_k g}$$

case 2 $v_f = \frac{1}{3}v_i$

$$\frac{1}{2}m\left(\frac{1}{3}v_i\right)^2 - \frac{1}{2}mv_i^2 = -\mu_k m g s'$$

$$\frac{1}{2}\left(-\frac{8}{9}v_i^2\right) = -\mu_k g s'$$

$$s' = \frac{\frac{8}{9}v_i^2}{2\mu_k g}$$

$$\frac{s'}{s} = \frac{\frac{\frac{8}{9}v_i^2}{2\mu_k g}}{\frac{v_i^2}{2\mu_k g}} = \frac{8}{9}$$

$$s' = \frac{8}{9}s \longleftarrow \text{"D"}$$

MC 3 $E_{k_{final}} - E_{k_{initial}} = -F_{Brake}\, d$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -F_{Brake}\, d$$

$V_f = 0$ (car comes to rest)

$$\frac{1}{2}mv_i^2 = F_{Brake}\, d$$

$$d = \frac{\frac{1}{2}mv_i^2}{F_{Brake}}$$

what if v_i is 2x greater?

$$d' = \frac{\frac{1}{2}m(2v_i)^2}{F_{Brake}}$$

$$= 4\left(\frac{\frac{1}{2}mv_i^2}{F_{Brake}}\right)$$

$$= 4d \longleftarrow \text{"A"}$$

7.14. Set Up: Use $K = \frac{1}{2}mv^2$ to relate v and K.
Solve: (a) $K = \frac{1}{2}(70\text{ kg})(32\text{ m/s})^2 = 3.6 \times 10^4\text{ J}$
(b) K is proportional to v^2, so K increases by a factor of 4 when v doubles.

7.19. Set Up: Use the work-kinetic energy theorem: $W_{net} = K_f - K_i = F_{net}s$. Since the net force is due to friction, $F_{net}s = -f_k s = -\mu_k mgs$. Also, since the car stops, $K_f = 0$.
Solve: (a) $W_{net} = K_f - K_i = F_{net}s$ gives $-\frac{1}{2}mv_i^2 = -\mu_k mgs$. Solving for the distance,

$$s = \frac{v_i^2}{2\mu_k g} = \frac{(23.0\text{ m/s})^2}{2(0.700)(9.80\text{ m/s}^2)} = 38.6\text{ m}$$

7.31. Set Up: Use $\Delta U_{grav} = mg(y_f - y_i)$.
Solve: $\Delta U_{grav} = (72\text{ kg})(9.80\text{ m/s}^2)(0.60\text{ m}) = 420\text{ J}$.
Reflect: This gravitational potential energy comes from elastic potential stored in his tensed muscles.

7.39. Set Up: The change in gravitational potential energy is $\Delta U_{grav} = mg(y_f - y_i)$, while the increase in kinetic energy, for a zero initial velocity, is $\Delta K = \frac{1}{2}mv_f^2$. Set the food energy, expressed in joules, equal to the mechanical energies developed.

Solve: **(a)** The food energy equals $mg(y_f - y_i)$, so

$$y_f - y_i = \frac{(140 \text{ food calories})(4186 \text{ J}/1 \text{ food calorie})}{(65 \text{ kg})(9.80 \text{ m/s}^2)} = 920 \text{ m}$$

(b) The mechanical energy would be 20% of the results of part (a): $\Delta y = (0.20)(920 \text{ m}) = 180 \text{ m}$.

7.40. Set Up: Set 20% of the food energy (in joules) equal to the total work you do in raising the weight N times.
Solve: For each arm raise, you do work equal to $W = mgh$ where $h = 35$ cm.

$$0.20(350 \text{ food cal})\left(\frac{4186 \text{ J}}{1 \text{ food cal}}\right) = N(mgh)$$

$$N = \frac{(0.20)(350)(4186 \text{ J})}{(5.0 \text{ kg})(9.80 \text{ m/s}^2)(0.35 \text{ m})} = 1.7 \times 10^4$$

Reflect: It is not reasonable to do this many repetitions in a single exercise session.

7.41. Set Up: Set 20% of the food energy (in joules) equal to the total work you do in jumping upward a distance of $h = 50$ cm, N times.
Solve:

$$0.20(500 \text{ food cal})\left(\frac{4186 \text{ J}}{1 \text{ food cal}}\right) = N(mgh)$$

$$N = \frac{(0.20)(500)(4186 \text{ J})}{(75 \text{ kg})(9.80 \text{ m/s}^2)(0.50 \text{ m})} = 1.1 \times 10^3 \text{ jumps}$$

The total time it would take is thus $N(2.0 \text{ s}) = 2.2 \times 10^3 \text{ s} = 37$ minutes.
Reflect: This would be a very strenuous workout even for a very fit person.

7.44. Set Up: Use $K_f + U_f = K_i + U_i$. Setting $y_i = h$, $y_f = 0$, $K_i = 0$ and $K_f = \frac{1}{2}mv_f^2$ gives $\frac{1}{2}mv_f^2 = mgh$, or $v_f = \sqrt{2gh}$.
Solve: **(a)** $v_f = \sqrt{2(9.80 \text{ m/s}^2)(500 \text{ m})} = 99 \text{ m/s} = 220$ mph
(b) Reflect: Hailstones actually have a much smaller speed when they reach the ground. Most of their initial potential energy is converted to thermal energy by the negative work done on them by the air drag force.

7.47. Set Up: Use $K_f + U_f = K_i + U_i$. Let $y_i = 0$ and $y_f = h$ and note that $U_i = 0$ while $K_f = 0$ at the maximum height. Consequently, conservation of energy becomes $mgh = \frac{1}{2}mv_i^2$.
Solve: **(a)** $v_i = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(0.20 \text{ m})} = 2.0 \text{ m/s}$.
(b) $K_i = mgh = (0.50 \times 10^{-6} \text{ kg})(9.80 \text{ m/s}^2)(0.20 \text{ m}) = 9.8 \times 10^{-7} \text{ J}$

$$\frac{K_i}{m} = \frac{9.8 \times 10^{-7} \text{ J}}{0.50 \times 10^{-6} \text{ kg}} = 2.0 \text{ J/kg}$$

(c) The human can jump to a height of

$$h_h = h_f\left(\frac{l_h}{l_f}\right) = (0.20 \text{ m})\left(\frac{2.0 \text{ m}}{2.0 \times 10^{-3} \text{ m}}\right) = 200 \text{ m}.$$

To attain this height, he would require a takeoff speed of: $v_i = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(200 \text{ m})} = 63 \text{ m/s}$.
(d) The human's kinetic energy per kilogram,

$$\frac{K_i}{m} = gh = (9.80 \text{ m/s}^2)(0.60 \text{ m}) = 5.9 \text{ J/kg}.$$

(e) Reflect: The flea stores the energy in its tensed legs.

8. MC 1

A, D.

A. Because of Newton's Third Law.

D. Because of the conservation of momentum.

8. MC 3

A, B, C.

Elastic collision ➔ kinetic energy conserved. So we have 2 equations (those for kinetic energy conservation and momentum conservation) and 2 unknowns (final velocities of the particles). The rules of algebra allow us to solve for *n* unknowns, if we have *n* equations. Once we have the final velocities (A), we can calculate the final kinetic energies (B) and momenta (C).

8. MC 5

C.

Use conservation of momentum.

momentum before firing = 0.

momentum after firing = $m_R v_R + m_B v_B$ =0

So $$v_R = \frac{-m_B}{m_R} v_B$$

8. MC 9

B.

At maximum compression the spring becomes an incompressible object connecting the two carts. Hence there velocities must be the same.

8.9. Set Up: $m_E = 5.98 \times 10^{24}$ kg. Consider the person and the earth to be an isolated system. Use coordinates where $+y$ is upward, in the direction the person jumps.

Solve: $P_{i,y} = P_{f,y}$. $P_{i,y} = 0$. The earth recoils in the $-y$ direction with speed v_E, so $0 = m_{person}v_{person} - m_E v_E$.

$$v_E = \left(\frac{m_{person}}{m_E}\right)v_{person} = \left(\frac{75\text{ kg}}{5.98 \times 10^{24}\text{ kg}}\right)(2.0\text{ m/s}) = 2.5 \times 10^{-23}\text{ m/s}$$

8.11. Set Up: Use coordinates where $+x$ is in the direction of the motion of the ball before it is caught. Figure 8.11 gives before and after sketches for the system of ball plus player. $(v_{b,i})_x = 100 \text{ mph} = 44.7 \text{ m/s}$

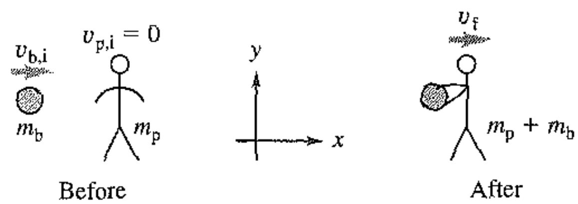

Figure 8.11

Solve: There is no external force in the x-direction, so $P_{i,x} = P_{f,x}$. This gives $m_b v_{b,i} = (m_b + m_p)v_f$.

$$v_f = \left(\frac{m_b}{m_b + m_p}\right)v_{b,i} = \left(\frac{0.145 \text{ kg}}{0.145 \text{ kg} + 65 \text{ kg}}\right)(44.7 \text{ m/s}) = 9.9 \text{ cm/s}$$

Reflect: There is a large increase in the mass that is moving so to maintain constant total momentum there is a large decrease in speed.

8.15. Set Up: Let $+x$ be east. Momentum is conserved in the east-west direction. Momentum is not conserved in the north-south direction because of the horizontal external force exerted by the tracks.

Solve: (a) The 25.0 kg mass retains its eastward velocity and momentum, the velocity of the handcar doesn't change. The final velocity of the handcar is 5.00 m/s, east.

(b) The mass has velocity zero relative to the earth. $P_{i,x} = P_{f,x}$ says $(200.0 \text{ kg})(5.00 \text{ m/s}) = (175.0 \text{ kg})v_{f,x}$ and $v_{f,x} = 5.71 \text{ m/s}$. The final velocity of the handcar is 5.71 m/s, east.

(c) $P_{i,x} = P_{f,x}$ says $(200.0 \text{ kg})(5.00 \text{ m/s}) + (25.0 \text{ kg})(-6.00 \text{ m/s}) = (225.0 \text{ kg})v_{f,x}$ and $v_{f,x} = 3.78 \text{ m/s}$. The final velocity of the handcar is 3.78 m/s, east.

(d) As in part (a) the 25.0 kg mass retains its eastward velocity and momentum. The velocity of the handcar doesn't change. The final velocity of the handcar is 5.00 m/s, east.

Reflect: By Newton's third law, the force exerted on the handcar by the mass that is thrown is opposite in direction to the force on the mass. In parts (a) and (d), these forces aren't parallel to the tracks and don't change the speed of the handcar. In part (b), the force on the mass is west so the reaction force on the handcar is east and the handcar speeds up. In part (c), the handcar exerts a force on the mass that is east, so the reaction force on the handcar is west and the handcar slows down.

8.16. Set Up: Let $+x$ be the direction the car is moving initially. Before it lands in the car the rain has no momentum along the x axis.

Solve: (a) $P_{i,x} = P_{f,x}$ says $(24{,}000 \text{ kg})(4.00 \text{ m/s}) = (27{,}000 \text{ kg})v_{f,x}$ and $v_{f,x} = 3.56 \text{ m/s}$.

(b) After it lands in the car the water must gain horizontal momentum, so the car loses horizontal momentum.

8.21. Set Up: Let x be the direction of motion. Let each boxcar have mass m.

Solve: (a) $P_{i,x} = P_{f,x}$ says $(3m)(20.0 \text{ m/s}) = (4m)v_{f,x}$ and $v_{f,x} = 15.0 \text{ m/s}$.

(b) $K_i = \frac{1}{2}(3m)(20.0 \text{ m/s})^2 = 600m \text{ J/kg}$; $K_f = \frac{1}{2}(4m)(15.0 \text{ m/s})^2 = 450m \text{ J/kg}$

$$\Delta K = -150m \text{ J/kg and } \frac{\Delta K}{K_i} = \frac{-150m \text{ J/kg}}{600m \text{ J/kg}} = -0.250;$$

25% of the original kinetic energy is dissipated. Kinetic energy is converted to other forms by work done by the forces during the collision.

8.33. Set Up: For an elastic collision with B initially stationary, the final velocities are

$$v_A = \left(\frac{m_A - m_B}{m_A + m_B}\right)v \text{ and } v_B = \left(\frac{2m_A}{m_A + m_B}\right)v.$$

Apply these equations with $m_A = 1.67 \times 10^{-27}\,\text{kg}$, $m_B = 6.65 \times 10^{-27}\,\text{kg}$ and $v = 258\,\text{km/s}$.

Solve: (a) $v_A = \left(\frac{1.67 \times 10^{-27}\,\text{kg} - 6.65 \times 10^{-27}\,\text{kg}}{1.67 \times 10^{-27}\,\text{kg} + 6.65 \times 10^{-27}\,\text{kg}}\right)(258\,\text{km/s}) = -154\,\text{km/s}$

$$v_B = \left(\frac{2[1.67 \times 10^{-27}\,\text{kg}]}{1.67 \times 10^{-27}\,\text{kg} + 6.65 \times 10^{-27}\,\text{kg}}\right)(258\,\text{km/s}) = 104\,\text{km/s}$$

The proton recoils to the left at 154 km/s and the alpha particle travels to the right at 104 km/s.

(b) The proton has initial kinetic energy

$$K_{A,\text{i}} = \tfrac{1}{2}m_A v^2 = \tfrac{1}{2}(1.67 \times 10^{-27}\,\text{kg})(2.58 \times 10^5\,\text{m/s})^2 = 5.56 \times 10^{-17}\,\text{J}$$

and final kinetic energy

$$K_{A,\text{f}} = \tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}(1.67 \times 10^{-27}\,\text{kg})(1.54 \times 10^5\,\text{m/s})^2 = 1.98 \times 10^{-17}\,\text{J}.$$

The kinetic energy lost is $K_{A,\text{i}} - K_{A,\text{f}} = 3.58 \times 10^{-17}\,\text{J}$.

(c) The kinetic energy gained by the alpha particle is

$$\tfrac{1}{2}(6.65 \times 10^{-27}\,\text{kg})(1.04 \times 10^5\,\text{m/s})^2 = 3.60 \times 10^{-17}\,\text{J}.$$

The energy lost by the proton is gained by the alpha particle. The total kinetic energy of the system is constant and the collision is elastic.

8.37. Set Up: Assume the ball is initially moving to the right, and let this be the $+x$ direction. The ball stops, so its final velocity is zero.

Solve: (a) $J_x = mv_{\text{f},x} - mv_{\text{i},x} = 0 - (0.145\,\text{kg})(36.0\,\text{m/s}) = -5.22\,\text{kg}\cdot\text{m/s}$ The magnitude of the impulse applied to the ball is $5.22\,\text{kg}\cdot\text{m/s}$.

(b) $J_x = F_x\,\Delta t$ so $F_x = \frac{J_x}{\Delta t} = \frac{-5.22\,\text{kg}\cdot\text{m/s}}{20.0 \times 10^{-3}\,\text{s}} = -261\,\text{N}$

Reflect: The signs of J_x and F_x show that both these quantities are to the left.