HOMEWORK 4 SOLUTIONS

CHAPTER 9

9.C1

Tangential acceleration of a point on a rotating object is the component of point's acceleration vector that is perpendicular to the radial component. Unlike the radial acceleration, the tangential acceleration is zero for a uniformly rotating object (i.e one whose angular velocity is constant).

9. MC1

B. One turn is 2π radians = 2 x 3.14 = 6.28 $\approx$ 6.

9. MC2

Angular motion is mathematically similar to linear motion. In particular have for the angular "distance" θ that an object with angular acceleration α rotates through at time t is

$$\theta = \tfrac{1}{2}\alpha t^2 \qquad (1)$$

(assuming that objects starts with zero angular velocity at angle of zero) [This is analogous to $x = \frac{1}{2}at^2$ for linear motion.]

We know that $t = 2T$ from the problem. All we need now is α. Have

$$\alpha = \frac{\omega_{final} - \omega_{initial}}{T}$$ The change in angular acceleration over the first revolution (2)

$$\omega_{average} = \frac{\omega_{final} + \omega_{initial}}{2}$$ The definition of average angular velocity (3)

We are told that $\omega_{initial} = 0$.

We are told that $\omega_{average} = \frac{1 revolution}{T}$.

Inserting this into (3) gives $\omega_{final} = \frac{2 revolutions}{T}$.

Inserting this into (2) gives $\alpha = \frac{2 revolutions}{T^2}$

Inserting this into (1) gives $\theta = \frac{1}{2}\left(\frac{2 revolutions}{T^2}\right)(2T)^2$ = 4 revolutions. **[C]**

9. MC10

From the definition of the moment of inertial I

$$I = m(2d)^2 + (2m)d^2 = 6md^2$$

After the masses switch positions, the new moment of inertial I' is

$$I' = (2m)(2d)^2 + md^2 = 9md^2$$

Solve first equation for *d*: $d = \sqrt{\frac{I}{6m}}$.

Insert this into the second equation:

$$I' = 9m\left(\sqrt{\frac{I}{6m}}\right)^2 = \frac{3}{2}I$$

[C]

9. MC11

The general formula for moment of inertial I is

$$I = \sum_i m_i r_i^2 .$$

If the r_i s double, I increases by a factor of 4. If the m_i s double, I increase by a factor of 2.
4 x 2 = 8.

[C]

9. MC15

Let $I = \beta MR^2$ where $\beta = \frac{2}{5}$ for the solid sphere, and $\beta = \frac{2}{3}$ for the hollow sphere.

Initial potential energy completely converted to kinetic energy.

Have energy conservation

initial potential energy = (final kinetic energy of linear motion) +
(final kinetic energy of rotational motion)

$$Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2$$

For rotational motion have $v = R\omega$. Also, $I = \beta MR^2$
So

$$Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\left(\beta MR^2\right)\left(\frac{v}{R}\right)^2$$

$$= \tfrac{1}{2}(1+\beta)Mv^2$$

Solve for v

$$v = \sqrt{\frac{2gh}{1+\beta}}$$

The larger is β, the smaller is v. So the hollow sphere ($\beta = \frac{2}{3}$) is moving more slowly than the solid sphere ($\beta = \frac{2}{5}$) at the bottom of the ramp.

Answer is **A.** The solid sphere is faster.

CHAPTER 10

10.1. Set Up: Let counterclockwise torques be positive. $\tau = Fl$ with $l = r\sin\phi$.
Solve: **(a)** $\tau = +(10.0\ \text{N})(4.00\ \text{m})\sin 90.0^\circ = 40.0\ \text{N}\cdot\text{m}$, counterclockwise.
(b) $\tau = +(10.0\ \text{N})(4.00\ \text{m})\sin 60.0^\circ = 34.6\ \text{N}\cdot\text{m}$, counterclockwise.
(c) $\tau = +(10.0\ \text{N})(4.00\ \text{m})\sin 30.0^\circ = 20.0\ \text{N}\cdot\text{m}$, counterclockwise.
(d) $\tau = -(10.0\ \text{N})(2.00\ \text{m})\sin 60.0^\circ = -17.3\ \text{N}\cdot\text{m}$, clockwise.
(e) $\tau = 0$ since the force acts on the axis and $l = 0$
(f) $\tau = 0$ since the line of action of the force passes through the location of the axis and $l = 0$.
Reflect: The torque of a force depends on the direction of the force and where it is applied to the object.

10.3. Set Up: Let counterclockwise torques be positive. $\tau = Fl$
Solve: $\tau_1 = -F_1R = -(7.50\ \text{N})(0.330\ \text{m}) = -2.48\ \text{N}\cdot\text{m}$.

$$\tau_2 = +F_2R = -(5.30\ \text{N})(0.330\ \text{m}) = +1.75\ \text{N}\cdot\text{m}.$$

$\sum\tau = \tau_1 + \tau_2 = -0.73\ \text{N}\cdot\text{m}$. The net torque is $0.73\ \text{N}\cdot\text{m}$, clockwise.

10.5. Set Up: Let counterclockwise torques be positive. $\tau = Fl$
Solve: $\tau_1 = -F_1l_1 = -(18.0\ \text{N})(0.090\ \text{m}) = -1.62\ \text{N}\cdot\text{m}$. $\tau_2 = +F_2l_2 = +(26.0\ \text{N})(0.090\ \text{m}) = +2.34\ \text{N}\cdot\text{m}$.
$\tau_3 = +F_3l_3 = +(14.0\ \text{N})(0.127\ \text{m}) = +1.78\ \text{N}\cdot\text{m}$.
$\sum\tau = \tau_1 + \tau_2 + \tau_3 = 2.50\ \text{N}\cdot\text{m}$, counterclockwise.
Reflect: It is important to take into account the direction of each torque when computing the net torque.

10.12. Set Up: Apply $\sum F_y = ma_y$ to the suitcase. Let $+y$ be downward. Apply $\sum \tau = I\alpha$ to the wheel. Let the counterclockwise sense of rotation be positive. The angular velocity ω and angular acceleration α of the wheel are related to the linear velocity v and linear acceleration a of the suitcase by $v = R\omega$ and $a = R\alpha$.

Solve: **(a)** $\omega = \dfrac{v}{R} = \dfrac{3.50\text{ m/s}}{0.400\text{ m}} = 8.75\text{ rad/s}$

(b) For the suitcase, $y - y_0 = 4.00\text{ m}$, $v_{0y} = 0$, $v_y = 3.50\text{ m/s}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{(3.50\text{ m/s})^2 - 0}{2(4.00\text{ m})} = +1.53\text{ m/s}^2.$$

$$\alpha = \frac{a_y}{R} = \frac{1.53\text{ m/s}^2}{0.400\text{ m}} = 3.82\text{ rad/s}^2.$$

The free-body diagram for the suitcase is given in Figure 10.12a.

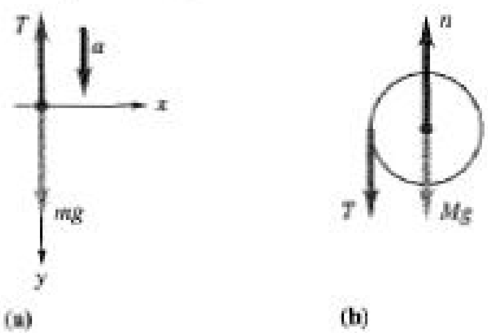

Figure 10.12

$\sum F_y = ma_y$ gives $mg - T = ma$ and $T = m(g - a) = (15.0\text{ kg})(9.80\text{ m/s}^2 - 1.53\text{ m/s}^2) = 124\text{ N}$. The free-body diagram for the wheel is given in Figure 10.12b. $\sum \tau = I\alpha$ gives $TR = I\alpha$ and

$$I = \frac{TR}{\alpha} = \frac{(124\text{ N})(0.400\text{ m})}{3.82\text{ m/s}^2} = 13.0\text{ kg}\cdot\text{m}^2$$

10.13. Set Up: For the pulley $I = \frac{1}{2}MR^2$. The elevator has

$$m_1 = \frac{22{,}500\text{ N}}{9.80\text{ m/s}^2} = 2300\text{ kg}.$$

The free-body diagrams for the elevator, the pulley and the counterweight are given in Figure 10.13. Apply $\sum \vec{F} = m\vec{a}$ to the elevator and to the counterweight. For the elevator take $+y$ upward and for the counterweight take $+y$ downward, in each case in the direction of the acceleration of the object. Apply $\sum \tau = I\alpha$ to the pulley, with clockwise as the positive sense of rotation. n is the normal force applied to the pulley by the axle. The elevator and counterweight each have acceleration a. $a = R\alpha$.

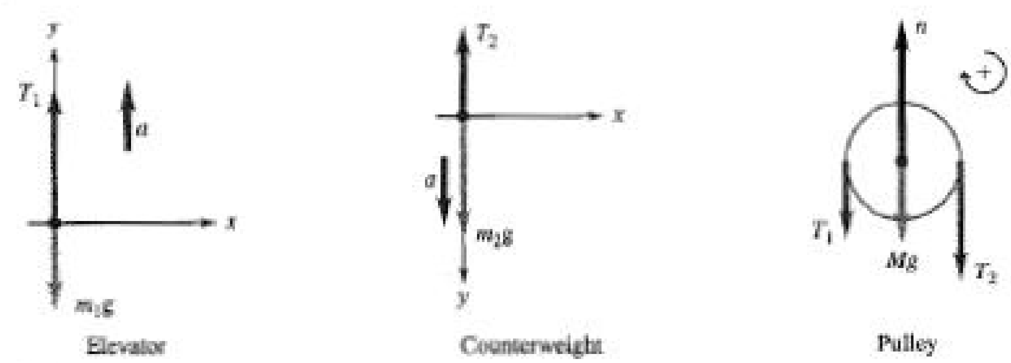

Figure 10.13

10.29. Set Up: For a thin-walled hollow cylinder $I = mR^2$. For a slender rod rotating about an axis through its center, $I = \frac{1}{12}ml^2$.

Solve: $L_i = L_f$ so $I_i\omega_i = I_f\omega_f$. $I_i = 0.40\ \text{kg}\cdot\text{m}^2 + \frac{1}{12}(8.0\ \text{kg})(1.8\ \text{m})^2 = 2.56\ \text{kg}\cdot\text{m}^2$. $I_f = 0.40\ \text{kg}\cdot\text{m}^2 + (8.0\ \text{kg})(0.25\ \text{m})^2 = 0.90\ \text{kg}\cdot\text{m}^2$.

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{2.56\ \text{kg}\cdot\text{m}^2}{0.90\ \text{kg}\cdot\text{m}^2}\right)(0.40\ \text{rev/s}) = 1.14\ \text{rev/s}$$

10.39. Set Up: The free-body diagram is given in Figure 10.39. F_f is the force on each foot and F_h is the force on each hand. Use coordinates as shown. Take the pivot at his feet and let counterclockwise torques to be positive.

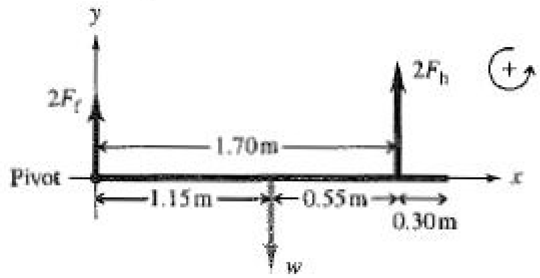

Figure 10.39

Solve: $\sum\tau = 0$ gives $+(2F_h)(1.70\ \text{m}) - w(1.15\ \text{m}) = 0$

$$F_h = w\frac{1.15\ \text{m}}{2(1.70\ \text{m})} = 0.338w = 272\ \text{N}$$

$\sum F_y = 0$ gives $2F_f + 2F_h - w = 0$ and $F_f = \frac{1}{2}w - F_h = 402\ \text{N} - 272\ \text{N} = 130\ \text{N}$

Reflect: His center of mass is closer to his hands than to his feet, so his hands exert a greater force.

Chapter 11 Multiple Choice

3. $$\text{Pressure} = \frac{F_\perp}{A} = \frac{F \sin 60^\circ}{2\ m^2}$$

$$= \frac{(10N)\frac{\sqrt{3}}{2}}{2\ m^2}$$

$$= \frac{5}{2}\sqrt{3}\ N/m^2$$

$$= 4.33\ N/m^2$$

$$= 4.33\ Pa \quad (B)$$

4. $$\text{Shear Stress} = \frac{F_\parallel}{A} = \frac{F \cos 60^\circ}{2\ m^2}$$

$$= \frac{(10N)\frac{1}{2}}{2\ m^2}$$

$$= 2.5\ Pa \quad (A)$$

9. Dimensions same, weight same $\Rightarrow$ stress is same

$$\text{stress} = -Y_A \frac{\Delta l_A}{l} = -Y_B \frac{\Delta l_B}{l}$$

$$-Y_A \frac{2\Delta l_B}{l} = -Y_B \frac{\Delta l_B}{l} \qquad \Delta l_A = 2\Delta l_B$$

$$Y_A = \frac{1}{2} Y_B \quad (C)$$

Chapter 11 Problems

11.5. Set Up: $A = 50.0\text{ cm}^2 = 50.0 \times 10^{-4}\text{ m}^2$. $Y = \dfrac{l_0 F_\perp}{A\,\Delta l}$

Solve: relaxed: $Y = \dfrac{(0.200\text{ m})(25.0\text{ N})}{(50.0 \times 10^{-4}\text{ m}^2)(3.0 \times 10^{-2}\text{ m})} = 3.33 \times 10^4\text{ Pa}$

maximum tension: $Y = \dfrac{(0.200\text{ m})(500\text{ N})}{(50.0 \times 10^{-4}\text{ m}^2)(3.0 \times 10^{-2}\text{ m})} = 6.67 \times 10^5\text{ Pa}$

11.8. Set Up: $Y = \dfrac{\text{stress}}{\text{strain}}$. A 5.0% elongation means $\Delta l/l_0 = 0.050$. For a spring, $F_T = kx$.

Solve: (a) stress $= Y \times$ strain $= (1474 \times 10^6\text{ Pa})(0.050) = 7.4 \times 10^7\text{ Pa}$

(b) stress $= F_T/A$ so $F_T =$ stress $\times A = (7.37 \times 10^7\text{ Pa})(78.1 \times 10^{-6}\text{ m}^2) = 5.76 \times 10^3\text{ N}$

The change in length is $x = \Delta l = (0.050)(25\text{ cm}) = 1.25\text{ cm}$. $F_T = kx$ gives

$$k = \frac{F_T}{x} = \frac{5.76 \times 10^3\text{ N}}{1.25 \times 10^{-2}\text{ m}} = 4.6 \times 10^5\text{ N/m}$$

(c) $F = 13mg = 13(75\text{ kg})(9.80\text{ m/s}^2) = 9555\text{ N}$ and $x = \dfrac{F_T}{k} = \dfrac{9555\text{ N}}{4.6 \times 10^5\text{ N/m}} = 2.08\text{ cm}$

11.9. Set Up: $Y = \dfrac{F_T/A}{\Delta l/l_0}$ so $F_T = \left(\dfrac{YA}{l_0}\right)\Delta l$ and $k = \dfrac{YA}{l_0}$. From Problem 11.8, $k = 4.6 \times 10^5\text{ N/m}$ for the natural Achilles tendon. $A = \pi r^2$

Solve: (a) $k = \dfrac{YA}{l_0}$ so $A = \dfrac{kl_0}{Y} = \dfrac{(4.6 \times 10^5\text{ N/m})(0.25\text{ m})}{30 \times 10^9\text{ Pa}} = 3.8 \times 10^{-6}\text{ m}^2$

$A = \pi r^2$ so $r = \sqrt{A/\pi} = 1.1$ mm and the diameter is 2.2 mm.

(b) The natural tendon has $r = \sqrt{(78.1\text{ mm}^2)/\pi} = 4.99$ mm and diameter 10.0 mm. The artificial tendon's diameter is much smaller.

Reflect: The artificial tendon has a larger Y and therefore a smaller diameter.

11.10. Set Up: stress $= \dfrac{F_\perp}{A}$, strain $= \dfrac{\Delta l}{l_0}$.

Solve: (a) $196 \times 10^6\text{ Pa} = \dfrac{F_\perp}{\pi(25 \times 10^{-6}\text{ m})^2}$ and $F_\perp = 0.385$ N.

(b) $0.40 = \dfrac{l - l_0}{l_0}$ and $l = 12$ cm gives $l_0 = 8.6$ cm.

11.11. Set Up: $= \dfrac{F_T/A}{\Delta l/l_0}$

Solve: (a) $F_T = 8mg = 5880$ N. $\dfrac{\Delta l}{l_0} = \dfrac{F_T}{AY} = \dfrac{5880\text{ N}}{(10 \times 10^{-4}\text{ m}^2)(24 \times 10^6\text{ Pa})} = 0.24 = 24\%$

(b) $F_T = 4mg$ so $\dfrac{\Delta l}{l_0} = \frac{1}{2}(24\%) = 12\%$

Reflect: Young's modulus for cartilage is much smaller than typical values for metals and the fractional change in length is larger.

11.14. Set Up: $\frac{\Delta V}{V_0} = -\frac{\Delta p}{B}$. $\Delta p = (1.0 \times 10^4\ \text{Pa/m})d$, where d is the depth below the surface.

Solve: (a) $\Delta V = -\frac{\Delta p}{B}V_0 = -\frac{(1.0 \times 10^4\ \text{Pa/m})(33\ \text{m})}{2.2 \times 10^9\ \text{Pa}}(1.0\ \text{cm}^3) = -1.5 \times 10^{-4}\ \text{cm}^3$

One cubic centimeter of her blood decreases in volume by $1.5 \times 10^{-4}\ \text{cm}^3$.

(b) $\frac{\Delta V}{V_0} = -\frac{1}{2}$ gives $\Delta p = B(\frac{1}{2}) = 1.1 \times 10^9\ \text{Pa}$

The depth would be $d = \frac{\Delta p}{1.0 \times 10^4\ \text{Pa/m}} = \frac{1.1 \times 10^9\ \text{Pa}}{1.0 \times 10^4\ \text{Pa/m}} = 1.1 \times 10^5\ \text{m} = 110\ \text{km}$. The ocean is not this deep; the greatest depth in the ocean is an order of magnitude less than this, about 11 km.

11.16. Set Up: $\frac{\Delta V}{V_0} = -\frac{\Delta p}{B}$. $1\ \text{atm} = 1.01 \times 10^5\ \text{Pa}$.

Solve: (a) $\Delta p = -B\frac{\Delta V}{V_0} = -(15 \times 10^9\ \text{Pa})(-0.0010) = 1.5 \times 10^7\ \text{Pa} = 150\ \text{atm}$

(b) The depth for a pressure increase of 1.5×10^7 Pa is 1.5 km. Unprotected dives do not approach this depth so bone compression is not a concern.