HOMEWORK 4 -- SOLUTIONS TO PROBLEMS

23.34. Set Up: $n = \frac{c}{v}$. The frequency of light doesn't change when it passes from one material into another. $v = f\lambda$. $\lambda = \frac{\lambda_0}{n}$. **Solve:** (a) $\lambda_v = \frac{\lambda_{0,v}}{n} = \frac{400 \text{ nm}}{1.34} = 299 \text{ nm}$. $\lambda_r = \frac{\lambda_{0,r}}{n} = \frac{700 \text{ nm}}{1.34} = 522 \text{ nm}$. Range is 299 nm to 522 nm. (b) Calculate the frequency in air, where $v = c = 3.00 \times 10^8 \text{ m/s}$. $f_r = \frac{c}{\lambda_r} = \frac{3.00 \times 10^8 \text{ m/s}}{700 \times 10^{-9} \text{ m}} = 4.29 \times 10^{14} \text{ Hz}$. $f_v = \frac{c}{\lambda_v} = \frac{3.00 \times 10^8 \text{ m/s}}{400 \times 10^{-9} \text{ m}} = 7.50 \times 10^{14} \text{ Hz}$. Range is $4.29 \times 10^{14} \text{ Hz}$ to $7.50 \times 10^{14} \text{ Hz}$. (c) $v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.34} = 2.24 \times 10^8 \text{ m/s}$

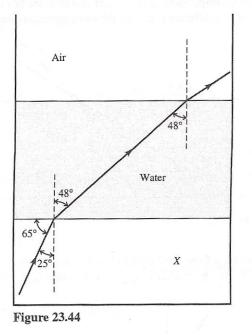
23.44. Set Up: Snell's law says $n_a \sin \theta_a = n_b \sin \theta_b$. Apply Snell's law to the refraction from material X into the water and then from the water into the air.

Solve: (a) material X to water: $n_a = n_X$, $n_b = n_w = 1.333$. $\theta_a = 25^\circ$ and $\theta_b = 48^\circ$.

$$n_a = n_b \left(\frac{\sin \theta_b}{\sin \theta_a} \right) = (1.333) \left(\frac{\sin 48^\circ}{\sin 25^\circ} \right) = 2.34$$

(b) water to air: As Figure 23.44 shows, $\theta_a = 48^\circ$. $n_a = 1.333$ and $n_b = 1.00$.

$$\sin\theta_b = \left(\frac{n_a}{n_b}\right) \sin\theta_a = (1.333) \sin 48^\circ = 82^\circ.$$



23.45. Set Up: $n_a \sin \theta_a = n_b \sin \theta_b$. The light is in diamond and encounters an interface with air, so $n_a = 2.42$ and $n_b = 1.00$. The largest θ_a is when $\theta_b = 90^\circ$.

Solve: (2.42) $\sin \theta_a = (1.00) \sin 90^\circ$. $\sin \theta_a = \frac{1}{2.42}$ and $\theta_a = 24.4^\circ$.

23.57. Set Up: For unpolarized light incident on a filter, $I = \frac{1}{2}I_0$ and the light is linearly polarized along the filter axis. For polarized light incident on a filter, $I = I_{max}(\cos \phi)^2$, where I_{max} is the intensity of the incident light, and the emerging light is linearly polarized along the filter axis.

Solve: (a) After the first filter, $I = \frac{1}{2}I_0$ and the light is polarized. After the second filter $I = (\frac{1}{2}I_0)(\cos 41.0^\circ)^2 = 0.285I_0$.

(b) The light is linearly polarized along the axis of the second filter.

24.MC 2

Answer is **A**. It must be a converging lens, because diverging lenses only produce virtual images.

24.5. Set Up: To find the location of the image use $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$. For a concave mirror R > 0. To find the height of the image use $m = \frac{y'}{y} = -\frac{s'}{s}$.

Solve: (a) s = 15.0 cm. $\frac{1}{s'} = \frac{2}{R} - \frac{1}{s} = \frac{2s - R}{Rs}$, so

$$s' = \frac{Rs}{2s - R} = \frac{(10.0 \text{ cm})(15.0 \text{ cm})}{30.0 \text{ cm} - 10.0 \text{ cm}} = 7.5 \text{ cm}.$$

 $m = -\frac{s}{s} = -\frac{7.5 \text{ cm}}{15.0 \text{ cm}} = -\frac{1}{2}$ and $y' = my = -\frac{1}{2}(8.00 \text{ mm}) = -4.00 \text{ mm}$. The image is 7.5 cm in front of the mirror. The image is 4.0 mm tall

and is inverted. (b) s = 10.0 cm. $s' = \frac{(10.0 \text{ cm})(10.0 \text{ cm})}{20.0 \text{ cm} - 10.0 \text{ cm}} = 10.0 \text{ cm}$. $m = -\frac{10.0 \text{ cm}}{10.0 \text{ cm}} = -1.00$. y' = my = -8.00 mm. The image is 10.0 cm in front of the mirror, at the location of the object. The image is 8.00 mm tall and is inverted.

(c) s = 2.50 cm. $s' = \frac{(10.0 \text{ cm})(2.50 \text{ cm})}{5.00 \text{ cm} - 10.0 \text{ cm}} = -5.00 \text{ cm}$. $m = -\frac{-5.00 \text{ cm}}{2.50 \text{ cm}} = +2.00$. y' = my = 16.0 mm. The

image is 5.00 cm behind the mirror. The image is 16.0 mm tall and is erect. (d) s = 1000.0 cm. $s' = \frac{(10.0 \text{ cm})(1000.0 \text{ cm})}{2000.0 \text{ cm} - 10.0 \text{ cm}} = +5.00$ cm. $m = -\frac{5.00 \text{ cm}}{1000.0 \text{ cm}} = -5.00 \times 10^{-3}$. y' = my = -0.040 mm. The image is 5.00 cm in front of the mirror. The image is 0.040 mm tall and is inverted.

Reflect: From $s' = \frac{Rs}{2s - R}$ we see that the image is real if s > R/2 and virtual if s < R/2. Real images are in front of the mirror and are inverted. Virtual images are behind the mirror and are erect.

24.6. Set Up: To find the location of the image use $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$. For a convex mirror R < 0. To find the height of the image use $m = \frac{y'}{y} = -\frac{s'}{s}$.

Solve: (a)
$$s = 15.0 \text{ cm}.$$
 $\frac{1}{s'} = \frac{2}{R} - \frac{1}{s} = \frac{2s - R}{Rs}, \text{ so } s' = \frac{Rs}{2s - R} = \frac{(-10.0 \text{ cm})(15.0 \text{ cm})}{30.0 \text{ cm} + 10.0 \text{ cm}} = -3.75 \text{ cm}.$
 $m = -\frac{s'}{s} = -\frac{3.75 \text{ cm}}{15.0 \text{ cm}} = +0.250.$

y' = my = +2.00 mm. The image is 3.75 cm behind the mirror. It is 2.00 mm tall and is upright. (b) s = 10.0 cm. $s' = \frac{(-10.0 \text{ cm})(10.0 \text{ cm})}{20.0 \text{ cm} + 10.0 \text{ cm}} = -3.33 \text{ cm}$. $m = -\frac{s'}{s} = -\frac{-3.33 \text{ cm}}{10.0 \text{ cm}} = +0.333$. y' = my = +2.67 mm. The image is 3.33 cm behind the mirror. It is 2.67 mm tall and is upright. (c) s = 2.50 cm. $s' = \frac{(-10.0 \text{ cm})(2.50 \text{ cm})}{5.00 \text{ cm} + 10.0 \text{ cm}} = -1.67$ cm. $m = -\frac{s'}{s} = -\frac{-1.67 \text{ cm}}{2.50 \text{ cm}} = +0.667$. y' = my = +5.33 mm. The image is 1.67 cm behind the mirror. It is 5.33 mm tall and is upright. (d) $s = 1000.0 \text{ cm.} s = \frac{(-10.0 \text{ cm})(1000.0 \text{ cm})}{2000.0 \text{ cm} + 10.0 \text{ cm}} = -4.98 \text{ cm.}$

$$m = -\frac{s}{s} = -\frac{-4.98 \text{ cm}}{1000.0 \text{ cm}} = +4.98 \times 10^{-3}$$

y' = my = +0.040 mm. The image is 4.98 cm behind the mirror. It is 0.040 mm tall and is upright.

24.9. Set Up: For a convex mirror, R < 0, so R = -18.0 cm and $f = \frac{R}{2} = -9.00$ cm.

Solve: (a)
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$
. $s' = \frac{sf}{s-f} = \frac{(1300 \text{ cm})(-9.00 \text{ cm})}{1300 \text{ cm} - (-9.00 \text{ cm})} = -8.94 \text{ cm}$.
 $m = -\frac{s'}{s} = -\frac{-8.94 \text{ cm}}{1300 \text{ cm}} = 6.88 \times 10^{-3}$. $|y'| = |m|y = (6.88 \times 10^{-3})(1.5 \text{ m}) = 0.0103 \text{ m} = 1.03 \text{ cm}$.

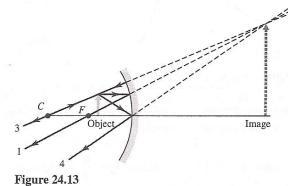
(b) The height of the image is much less than the height of the car, so the car appears to be farther away than its actual distance.

Reflect: Problem 24.11 shows that the image formed by a convex mirror is always virtual and smaller than the object.

24.13. Set Up: For a concave mirror, R > 0. R = 32.0 cm and $f = \frac{R}{2} = 16.0$ cm.

Solve: (a) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $s' = \frac{sf}{s-f} = \frac{(12.0 \text{ cm})(16.0 \text{ cm})}{12.0 \text{ cm} - 16.0 \text{ cm}} = -48.0 \text{ cm}$. $m = -\frac{s'}{s} = -\frac{-48.0 \text{ cm}}{12.0 \text{ cm}} = +4.00$. (b) s' = -48.0 cm, so the image is 48.0 cm to the right of the mirror. s' < 0 so the image is virtual.

(c) The principal-ray diagram is sketched in Figure 24.13. The rules for principal rays apply only to paraxial rays. Principal ray 2, that travels to the mirror along a line that passes through the focus, makes a large angle with the optic axis and is not described well by the paraxial approximation. Therefore, principal ray 2 is not included in the sketch.



Reflect: A concave mirror forms a virtual image whenever s < f.

24.24. Set Up: The image formed by refraction at the surface of the eye is located by $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $n_a = 1.00, n_b = 1.35, R > 0$. For a distant object, $s \approx \infty$ and $\frac{1}{s} \approx 0$.

Solve: (a) $s \approx \infty$ and s' = 2.5 cm: $\frac{1.35}{2.5 \text{ cm}} = \frac{1.35 - 1.00}{R}$ and R = 0.648 cm = 6.48 mm.

(b) R = 0.648 cm and s = 25 cm: $\frac{1.00}{25 \text{ cm}} + \frac{1.35}{s'} = \frac{1.35 - 1.00}{0.648}$. $\frac{1.35}{s'} = 0.500$ and s' = 2.70 cm = 27.0 mm. The image is formed behind the retina.

(c) Calculate s' for $s \approx \infty$ and R = 0.50 cm: $\frac{1.35}{s'} = \frac{1.35 - 1.00}{0.50 \text{ cm}}$. s' = 1.93 cm = 19.3 mm. The image is formed in front of the retina.

24.29. Set Up: $m = \frac{y'}{y} = -\frac{s'}{s}$. Since the image is erect, y' > 0 and m > 0. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. Solve: $m = \frac{y'}{y} = \frac{1.30 \text{ cm}}{0.400 \text{ cm}} = +3.25$. $m = -\frac{s'}{s} = +3.25$ gives s' = -3.25s. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives $\frac{1}{s} + \frac{1}{-3.25s} = \frac{1}{7.00 \text{ cm}}$ and s = 4.85 cm.

s' = -(3.25)(4.85 cm) = -15.8 cm. The object is 4.85 cm to the left of the lens. The image is 15.8 cm to the left of the lens and is virtual.

Reflect: The image is virtual because the object distance is less than the focal length.

24.37. Set Up: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. 1 is the surface closest to the object. *R* is positive if the center of curvature is on the side of the lens opposite the object. For a flat surface, $R \to \infty$.

Solve: (a)
$$L_1: \frac{1}{f} = (0.5) \left(\frac{1}{11.5 \text{ cm}} - \frac{1}{-10.5 \text{ cm}} \right)$$
 and $f = +11.0 \text{ cm}$.
 $L_2: \frac{1}{f} = (0.5) \left(\frac{1}{10.5 \text{ cm}} - \frac{1}{-11.5 \text{ cm}} \right)$ and $f = +11.0 \text{ cm}$.
(b) $L_1: \frac{1}{f} = (0.5) \left(\frac{1}{\infty} - \frac{1}{-8.50 \text{ cm}} \right)$ and $f = +17.0 \text{ cm}$.
 $L_2: \frac{1}{f} = (0.5) \left(\frac{1}{8.50 \text{ cm}} - \frac{1}{\infty} \right)$ and $f = +17.0 \text{ cm}$.
(c) $L_1: \frac{1}{f} = (0.5) \left(\frac{1}{9.80 \text{ cm}} - \frac{1}{11.5 \text{ cm}} \right)$ and $f = +133 \text{ cm}$.
 $L_2: \frac{1}{f} = (0.5) \left(\frac{1}{-9.80 \text{ cm}} - \frac{1}{-11.5 \text{ cm}} \right)$ and $f = -133 \text{ cm}$.
(d) $L_1: \frac{1}{f} = (0.5) \left(\frac{1}{-9.20 \text{ cm}} - \frac{1}{\infty} \right)$ and $f = -18.4 \text{ cm}$.
 $L_2: \frac{1}{f} = (0.5) \left(\frac{1}{\infty} - \frac{1}{9.20 \text{ cm}} \right)$ and $f = -18.4 \text{ cm}$.
 $L_2: \frac{1}{f} = (0.5) \left(\frac{1}{-10.4 \text{ cm}} - \frac{1}{11.6 \text{ cm}} \right)$ and $f = -11.0 \text{ cm}$.
 $L_2: \frac{1}{f} = (0.5) \left(\frac{1}{-11.6 \text{ cm}} - \frac{1}{10.4 \text{ cm}} \right)$ and $f = -11.0 \text{ cm}$.

24.39. Set Up: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. If *R* is the radius of the lens, then $R_1 = R$ and $R_2 = -R$. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $m = \frac{y'}{y} = -\frac{s'}{s}$.

Solve: (a) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(n-1)}{R}.$

$$R = 2(n-1)f = 2(0.44)(8.0 \text{ mm}) = 7.0 \text{ mm}.$$

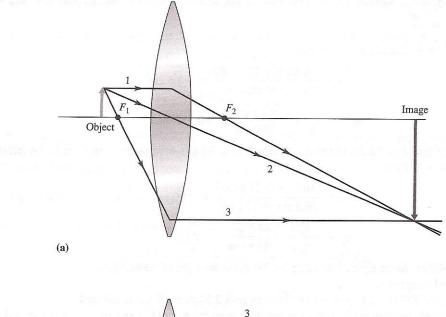
0.44 cm = 4.4 mm.

s' > 0 so the image is real. m < 0 so the image is inverted.

Reflect: The lens is converging and has a very short focal length. As long as the object is farther than 7.0 mm from the eye, the lens forms a real image.

24.49. Set Up: f = +14.0 cm. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives $s' = \frac{sf}{s-f}$. $m = \frac{y'}{y} = -\frac{s'}{s}$. Solve: s = 18.0 cm(a) $s' = \frac{sf}{s - f} = \frac{(18.0 \text{ cm})(14.0 \text{ cm})}{18.0 \text{ cm} - 14.0 \text{ cm}} = 63.0 \text{ cm}$. The image is 63.0 cm to the right of the lens. (b) $m = -\frac{s'}{s} = -\frac{63.0 \text{ cm}}{18.0 \text{ cm}} = -3.50$

(d) m < 0 so the image is inverted. The principal-ray diagram is sketched in Figure 24.49a.



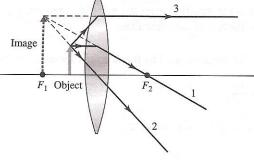


Figure 24.49

(b)

$$s = 7.00 \, \text{cm}$$

(e) $s' = \frac{sf}{s-f} = \frac{(7.00 \text{ cm})(14.0 \text{ cm})}{7.00 \text{ cm} - 14.0 \text{ cm}} = -14.0 \text{ cm}$. The image is 14.0 cm to the left of the lens. (f) $m = -\frac{s'}{s} = -\frac{-14.0 \text{ cm}}{7.00 \text{ cm}} = +2.00$

(g) s' < 0 so the image is virtual.

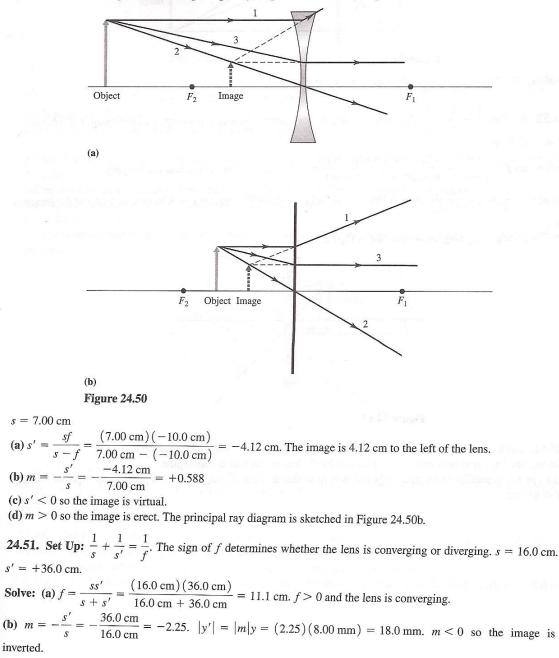
(h) m > 0 so the image is erect. The principal-ray diagram is sketched in Figure 24.49b.

Reflect: For a converging lens, when s > f the image is real and when s < f the image is virtual.

24.50. Set Up: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}$. f = -10.0 cm. Solve: s = 18.0 cm (a) $s' = \frac{sf}{s-f} = \frac{(18.0 \text{ cm})(-10.0 \text{ cm})}{18.0 \text{ cm} - (-10.0 \text{ cm})} = -6.43$ cm. The image is 6.43 cm to the left of the lens. (b) $m = -\frac{s'}{s} = -\frac{-6.43 \text{ cm}}{18.0 \text{ cm}} = +0.357$

(c) s' < 0 so the image is virtual.

(d) m > 0 so the image is erect. The principal ray diagram is sketched in Figure 24.50a.



25.20. Set Up: For an object 25.0 cm from the eye, the corrective lens forms a virtual image at the near point of the eye.

Solve: (a) The person is farsighted.

(b) A converging lens is needed.

(b) A converging lens is needed. (c) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $f = \frac{ss'}{s+s'} = \frac{(25.0 \text{ cm})(-45.0 \text{ cm})}{25.0 \text{ cm} - 45.0 \text{ cm}} = +56.2 \text{ cm}$. The power is $\frac{1}{0.562 \text{ m}} = +1.78$ diopters.

25.21. Set Up: For an object 25.0 cm from the eye, the corrective lens forms a virtual image at the near point of the eye. The distances from the corrective lens are s = 23.0 cm and s' = -43.0 cm.

Solve: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $f = \frac{ss'}{s+s'} = \frac{(23.0 \text{ cm})(-43.0 \text{ cm})}{23.0 \text{ cm} - 43.0 \text{ cm}} = +49.4 \text{ cm}$. The power is $\frac{1}{0.494 \text{ m}} = 2.02$ diopters. Reflect: In Problem 25.20 the contact lenses have power 1.78 diopters. The power of the lenses is different for ordinary glasses versus contact lenses.

26. MC 4

Submersion in water reduces the wavelength of light Answer is **C**.

(wavelength = wavelength_in_vacuum/index_of_refraction). The distances of the dark bands from the center spot are proportional to wavelength/slit_separation. Reducing the wavelength (through submersion) reduces this ratio, which causes the dark bands to move toward the center spot.

26. MC 8

Answer is A. Dark fringe separation ~ wavelength/slit width. So increasing slit width decreases distance between first two dark fringes.

26.3. Set Up: The nature of the interference depends on the path difference. Solve: (a) The person is a distance $r_A = D + d$ from A and a distance $r_B = d$ from B. The path difference is $r_A - r_B = D + d - d = D$. The interference is determined by D and is independent of d. (b) No, the loudness won't change. As he walks, the path difference remains constant.

26.9. Set Up: $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.32 \times 10^{14} \text{ Hz}} = 4.75 \times 10^{-7} \text{ m}$. Bright fringes are located at $y_m = R \frac{m\lambda}{d}$, when $y_m \ll R$. Dark fringes are at $d\sin\theta = (m + \frac{1}{2})\lambda$ and $y = R\tan\theta$. For the third bright fringe (not counting the central bright spot), m = 3. For the third dark fringe, m = 2.

Solve: (a) $d = \frac{m\lambda R}{y_m} = \frac{3(4.75 \times 10^{-7} \text{ m})(0.850 \text{ m})}{0.0311 \text{ m}} = 3.89 \times 10^{-5} \text{ m} = 0.0389 \text{ mm}$ (b) $\sin\theta = (2 + \frac{1}{2})\frac{\lambda}{d} = (2.5)\left(\frac{4.75 \times 10^{-7} \text{ m}}{3.89 \times 10^{-5} \text{ m}}\right) = 0.0305 \text{ and } \theta = 1.75^{\circ}. \ y = R\tan\theta = (85.0 \text{ cm})\tan 1.75^{\circ} = 1.75^{\circ}.$ 2.60 cm.

Reflect: The third dark fringe is closer to the center of the screen than the third bright fringe on one side of the central bright fringe.

26.24. Set Up: Both reflections occur for waves in the plastic substrate reflecting from the reflective coating, so they both have the same phase shift upon reflection and the condition for destructive interference (cancellation) is

 $2t = (m + \frac{1}{2})\lambda$, where t is the depth of the pit. $\lambda = \frac{\lambda_0}{n}$. The minimum pit depth is for m = 0.

Solve: $2t = \frac{\lambda}{2}$. $t = \frac{\lambda}{4} = \frac{\lambda_0}{4n} = \frac{790 \text{ nm}}{4(1.8)} = 110 \text{ nm} = 0.11 \ \mu\text{m}.$

26.25. Set Up: The condition for a dark fringe is $\sin \theta = \frac{m\lambda}{a}$, $m = \pm 1, \pm 2, \ldots$

Solve: $\sin\theta = m \left(\frac{632.8 \times 10^{-9} \text{ m}}{0.00375 \times 10^{-3} \text{ m}} \right) = m(0.1687). \ m = \pm 1: \ \theta = \pm 9.71^{\circ}. \ m = \pm 2: \ \theta = \pm 19.7^{\circ}. \ m = \pm 3: \ \theta = \pm 30.4^{\circ}. \ m = \pm 4: \ \theta = \pm 42.4^{\circ}. \ m = \pm 5: \ \theta = \pm 57.5^{\circ}.$

Reflect: There are a finite number of dark fringes because $\frac{m\lambda}{a} = \sin\theta$ can't be larger than 1.00. This establishes a maximum value for *m*.

26.35. Set Up: The bright fringes are located at angles θ given by $\sin \theta = \frac{m\lambda}{d}$. The separation d between adjacent slits in the grating is $d = \frac{1.00 \times 10^{-3} \text{ m}}{400.0} = 2.50 \times 10^{-6} \text{ m}$. Solve: $\sin \theta = m \left(\frac{600.0 \times 10^{-9} \text{ m}}{2.50 \times 10^{-6} \text{ m}} \right) = m(0.240)$. First-order (m = 1): $\theta = 13.9^{\circ}$. Second-order (m = 2): $\theta = 28.7^{\circ}$. Third-order (m = 3): $\theta = 46.1^{\circ}$.

26.43. Set Up: The maxima are at angles θ given by $2d\sin\theta = m\lambda$, where d is the spacing between adjacent planes in the crystal.

Solve: m = 2. $d = \frac{2\lambda}{2\sin\theta} = \frac{0.0850 \text{ nm}}{\sin 21.5^\circ} = 0.232 \text{ nm}$