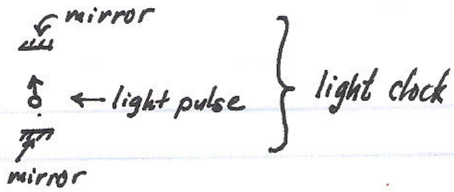
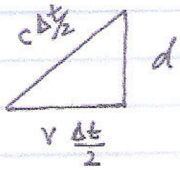
See this Light Clock Simulation to understand this calculation.



Time Dilation

Consider light clock moving with speed v



$$\frac{2d}{c} = \Delta t_p$$

$$d = \frac{c\,\Delta t_p}{2}$$

$$c^2\left(\frac{\Delta t}{2}\right)^2 = v^2\left(\frac{\Delta t}{2}\right)^2 + d^2$$

$$(c^2 - v^2)\left(\frac{\Delta t}{2}\right)^2 = d^2$$

$$= c^2\left(\frac{\Delta t_p}{2}\right)^2$$

$$\left(1 - \frac{v^2}{c^2}\right)(\Delta t)^2 = (\Delta t_p)^2$$ divide by c^2, mult by 4

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - v^2/c^2}}$$

$$= \gamma \Delta t_p$$

Δt_p = duration of a tick of stationary light clock

Δt = duration of a tick of moving light clock

tick = travel of light pulse from bottom mirror to top mirror and back

See this Length Contraction Tutorial to see the derivation of the formula below.

Length Contraction

$$L = \frac{L_0}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

L_0 = length of stationary object

L = length of moving object

#2

a) $v = 0.60\,c = \frac{3}{5} c$

$$\frac{L}{L_0} = \gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5} = 80\%$$

b) $v = 90\,km/hr = \dfrac{90\,000\,m}{3600\,s}$

$$= \frac{900}{36}\frac{m}{s} = \frac{100}{4}\frac{m}{s} = 25\ m/s$$

$$\frac{v}{c} = \frac{25\ m/s}{3 \times 10^8\,m/s} = \frac{25}{30 \times 10^7} = \frac{5}{6} \times 10^{-7}$$

$$= 8.3 \times 10^{-8}$$

$$\gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\boxed{\text{MATH FACT} \quad \sqrt{1+\epsilon} \approx 1 + \tfrac{1}{2}\epsilon \qquad \epsilon \ll 1}$$