Relativity

I. SPECIAL RELATIVITY

Inertial reference frame — a "laboratory" that is not accelerating

Principle of Relativity — the laws of physics are the same in all inertial reference frames

That light travels at 3×10^8 m/s in a vacuum is a law of physics

$\Rightarrow$ Light travels at the same speed in all inertial reference frames

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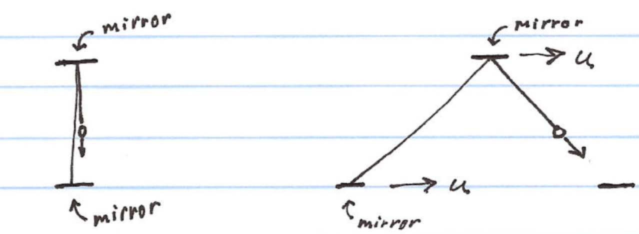
CONSEQUENCES

(1) Time Dilation — clock in motion seems to slow down

light clock — a photon (particle of light) bouncing between two mirrors



stationary

moving (position of mirror shown when photon in contact with them)

photon travels longer distance at <u>same speed</u> c.

So time for tick is longer, i.e. <u>clock slower</u>

Q. How much slower?

A. $$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \gamma \Delta t_0 \qquad \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Δt_0 = duration of tick when stationary

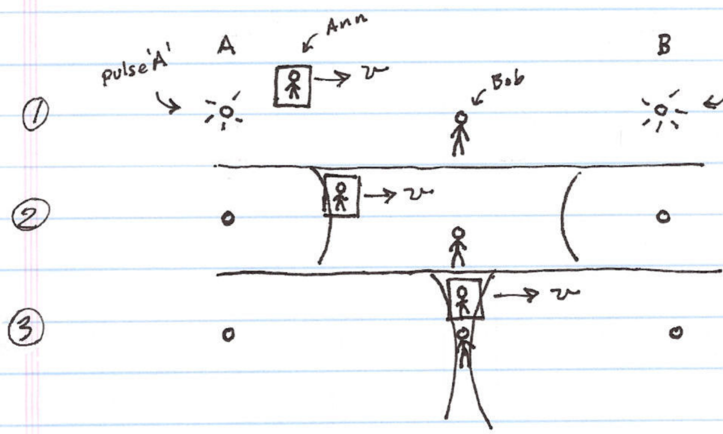
Δt = duration of tick of moving clock measured by stationary observer

u = speed of clock relative to stationary observer

c = speed of light

(2) Simultaneity is not absolute.

Events that seem to be simultaneous to a stationary observer, do not seem so to a moving observer (even when the moving observer is instantaneously at the location of the stationary observer when he observes the events)



Ann sees pulse 'A' first.
Bob has not yet seen pulses.

Bob sees pulses at the same time.

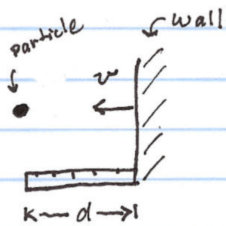
(3) Length Contraction

- Consider a particle moving toward a wall with speed v.
- In the frame of the particle, the wall is moving toward the particle with speed v
- In the frame of the particle, the wall will hit the particle at a time $\tau = d/v$, where d is the distance to the wall in the particle's frame.
- In the frame of the wall, the time τ is dilated. To an observer stationary with respect to the wall, the time τ' for the particle to hit the wall is $\tau' = \gamma \tau$.
- In the frame of the wall, the for the particle to hit the wall is given by $\tau' = \frac{l}{v}$, where l is the distance from the wall to the particle measured in the wall's frame
- Have

$$v = \frac{d}{\tau} = \frac{l}{\tau'} = \frac{l}{\gamma \tau}$$

$$\Rightarrow d = \frac{l}{\gamma}$$

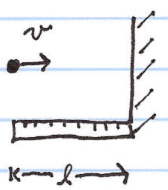
Length in moving frame is shorter than same length in stationary frame ($\gamma \geq 1$)



$\tau = \frac{d}{v}$

Frame of particle

Wall approaches particle at speed v



$\tau' = \gamma \tau$
$= \frac{l}{v}$

Frame of wall

Particle approaches wall at speed v

- Imagine a stick whose length is the distance between the particle and the wall. The length of the moving stick (d) is shorter than the length of the stationary stick (ℓ).

④ Equivalence of Mass and Energy

- In relativity time is just another dimension
- So the vectors that we used is classical physics are replaced by vectors that have a time component added

position $\quad x \rightarrow \begin{bmatrix} ct \\ x \end{bmatrix}$

velocity $\quad v_x \rightarrow \begin{bmatrix} v_t \\ v_x \end{bmatrix} = \begin{bmatrix} \gamma c \\ \gamma v_x \end{bmatrix}$

momentum $\quad p_x \rightarrow \begin{bmatrix} p_t \\ p_x \end{bmatrix} = \begin{bmatrix} E/c \\ p_x \end{bmatrix}$

- The way that the laws of physics remain the same, is that the lengths of these vectors is the same in every inertial reference frame
- The length of the new momentum vector is

$$m^2c^4 = E^2 - (pc)^2 = \text{length} \times c^2$$

use negative pythagorean theorem to get length

- In frame that $p = 0$,

$$\boxed{mc^2 = E}$$

Energy of a particle is proportional to its mass

⑤ Effective mass increase with speed

For $\vec{F} = \frac{d\vec{p}}{dt}$ to have the same form

in all inertial reference frames

$$\vec{p} = m_0 \gamma \vec{v}$$

$m_0 \gamma$ = "relativistic mass"

m_0 = "rest mass" (mass measured when particle is at rest)

Total Energy of moving particle

$$E = mc^2$$
$$= m_0 \gamma c^2$$

$$E_{rest} = m_0 c^2$$

Kinetic energy = Total Energy − Rest Energy

$$= m_0 \gamma c^2 - m_0 c^2$$
$$= m_0 c^2 (\gamma - 1)$$
$$= m_0 c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)$$
$$\approx m_0 c^2 \left(\frac{1}{1 - \frac{v^2}{2c^2}} - 1 \right) \qquad \frac{v}{c} \ll 1$$
$$\approx m_0 c^2 \left[\left(1 + \frac{v^2}{2c^2} \right) - 1 \right]$$
$$\approx \frac{1}{2} m_0 v^2 \longleftarrow$$

non-relativistic Kinetic energy

II. General Relativity

- Describes Gravity
- Mass bends space-time
- Particles move along shortest paths in bent spacetime
- "Matter tells spacetime how to bend,
  spacetime tells matter how to move."
- Black hole – case of extreme bending of spacetime.